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Research Article

**NUMERICAL MODELING OF INTERNAL TRANSMISSION  
DYNAMICS HEPATITIS C VIRUS**<sup>1</sup>Sahar Noor, <sup>2</sup>Dr. Baseera Imran, <sup>3</sup>M Hassaan ul Haq<sup>1</sup>MPhil In Mathematics, University of Lahore<sup>2</sup>MBBS MPhil in Physiology,<sup>3</sup>MS In Management in Sciences, University ofLahore.**Article Received:** April 2021**Accepted:** April 2021**Published:** May 2021**Abstract:**

*In this paper, a mathematical model for internal transmission dynamics of HCV is formulated, based on the four-population immunized populace, an inoculated uncovered populace, recovered individual and immune response T cells populace. The model is analyzed by explicit numerical scheme i.e. Non Standard Finite Difference (NSFD) scheme preserve the monotonicity of solution irrespective of step size. Result are compared with conventional numerical scheme like Forward Euler's method and RK-4. Unlike Forward Euler's and RK-4 methods are failed for large time step; the proposed NSFD scheme is not only independent of time size but also conserve the stability of continuous dynamical behavior.*

*Keywords: Epidemic, cirrhosis, hepatocellular carcinoma, T cells, threshold parameter*

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**INTRODUCTION:**

Hepatitis C virus (HCV) is a contagious virus that causes acute hepatitis and chronic liver diseases leading to liver cirrhosis and hepatocellular carcinoma [1]. It is a global epidemiology with over 170–200 million people having chronic hepatitis C virus (HCV) infection [2][3][4].

25% of hepatocellular carcinomas, 30% of all cases of cirrhosis and over 350,000 mortality rate are attributable to this infection [5] [6]. Hepatitis C Virus RNA is analyzed with the polymerase chain reaction (PCR) to examine the infectivity [7]. HCV genotype not only narrates the effects of treatment but also the history of disease. Mathematical models are formulated to understand immunologic response to virus and effectiveness of drug therapy.

Appropriate mathematical models can not only define the dynamics of immune response and the effectiveness of drug therapy but also provide biological answers with pathogenesis [8]. Our epidemic model possesses four variables: Healthy liver cells  $H_s$ , infected liver cells  $H_i$ , Virus load  $V$ , and immune response T cells. The few assumptions are considered as  $\beta_s$  is a constant rate for the forming of healthy liver cells,  $H_s$  and  $\mu_s$  is a constant rate for die out.  $H_s$  cells are infected at the rate proportional to the product of  $H_s$  and  $V$  with proportionality constant  $k$ ; and once infected die with the constant rate  $\mu_i$ . T cells eradicate infected cells  $H_i$  at the rate proportional to the product of  $H_i$  and  $T$  with proportionality constant  $\delta$ .

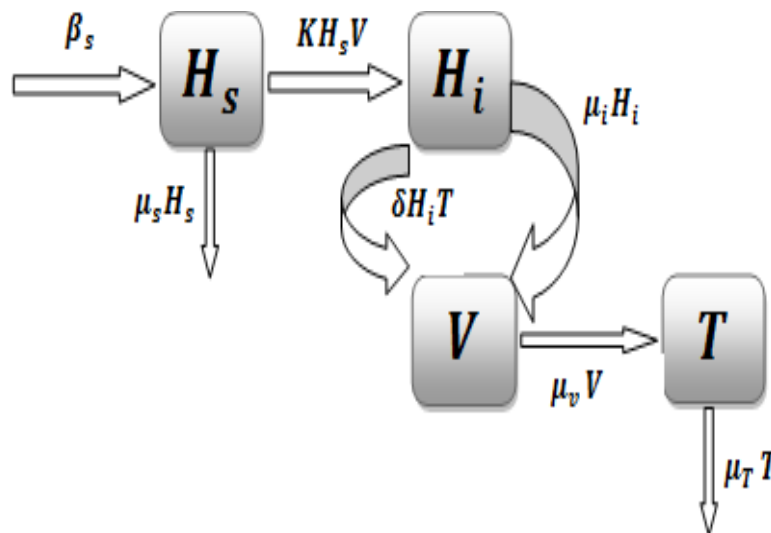
**Modified SIR Epidemic Model**

Figure 1: Compartmental model of internal dynamics of HCV Virus

Where,

Susceptible liver cells  
 Rate of growth of susceptible liver cells  
 Rate of death of susceptible liver cells  
 Infected liver cells  
 Rate of death of infected liver cells  
 Virus load  
 The response of cell on the dynamics of infection  
 Growth rate of T cells  
 Maximum T cell population level  
 Rate of death of T cells

### System of Differential Equation

The following differential equations are obtained from the above model.

Where, then epidemic will exist

### CONDITION FOR EPIDEMIC

is a basic reproductive number or threshold parameter for the proposed model.

### Equilibrium points of the model

The two equilibrium points of the epidemic model are  $E_1 = (I_0) =$  Disease Free Equilibrium(DFE) if and  $E_2 = (I_i) =$  Endemic Equilibrium (EE) if

### Locally Stability of DFE Equilibrium

To analyze the local stability of the DFE point i.e.  $E_1 = (I_0) =$

$$J = \begin{bmatrix} -KV - \mu_s & 0 & -KH_s & 0 \\ KV & -\delta T - \mu_i & KH_s & -\delta H_i \\ 0 & \rho & -\mu_v & 0 \\ 0 & 0 & \beta_T \left(1 - \frac{T}{T_{\max}}\right) & \frac{-\beta_T V}{T_{\max}} - \mu_T \end{bmatrix}$$

Clearly two Eigen values are . The two remaining two eigen values can be find out by

We know that if then, then the fixed point is stable.

Hence points are stable i.e.

### NUMERICAL ANALYSIS

NSFD scheme was first introduced by R.E. Mickens in 1989. In different areas numerous discretization of non-linear equations have been established. According to Mickens the NSFD is defined as “A non-standard finite variance scheme is any discrete depiction of a system of differential equations that is based on the given rules

#### NSFD Method

Let  $H^n$ ,  $H^n$ ,  $V^n$  and  $T^n$  denote the value of  $H_s$ ,  $H_i$ ,  $V$  and  $T$  at  $t=n$ . Using non-standard finitedifference Modeling theory system (2) can be written as follows

#### Convergence Analysis

The stability of the DFE point for the proposed NSFD scheme is discussed here, by considering the first equation of system (1)

$$F^* = \begin{bmatrix} \frac{1}{1 + \mu_s h} - \lambda & 0 & -\frac{\left(\frac{\beta_s}{\mu_s} + h\beta_s\right)(hK)}{(1 + \mu_s h)^2} & 0 \\ 0 & \frac{1}{1 + \mu_i h} - \lambda & \frac{hK \beta_s}{1 + \mu_i h} & 0 \\ 0 & \frac{\rho h}{1 + \mu_v h} & \frac{1}{1 + \mu_v h} - \lambda & 0 \\ 0 & 0 & \frac{h\beta_T}{1 + h\mu_T} & \frac{1}{1 + h\mu_T} - \lambda \end{bmatrix}$$

Clearly

v

**Lemma**

Let  $\lambda_1$  and  $\lambda_2$  be the Eigen values of Jacobian matrix J and A and B be the trace and the determinant of the same Jacobean, Then the absolute values of  $\lambda_1$  and  $\lambda_2$  are less than unity if;

I.  $1 - A + B > 0$

II.  $1 + A + B > 0$

III.  $B < 1$

(i)  $1 - A + B > 0$   
 $1 - +$

(ii)  $1 + A + B > 0$

(iii)  $B < 1$

Which is true

All values in nominator and denominator are positive as  $\lambda_1$  and  $\lambda_2$ . As all the condition of above lemma are satisfied Therefore, we can conclude that the DFE point is stable for all step sizes h whenever

**Numerical Experiments**

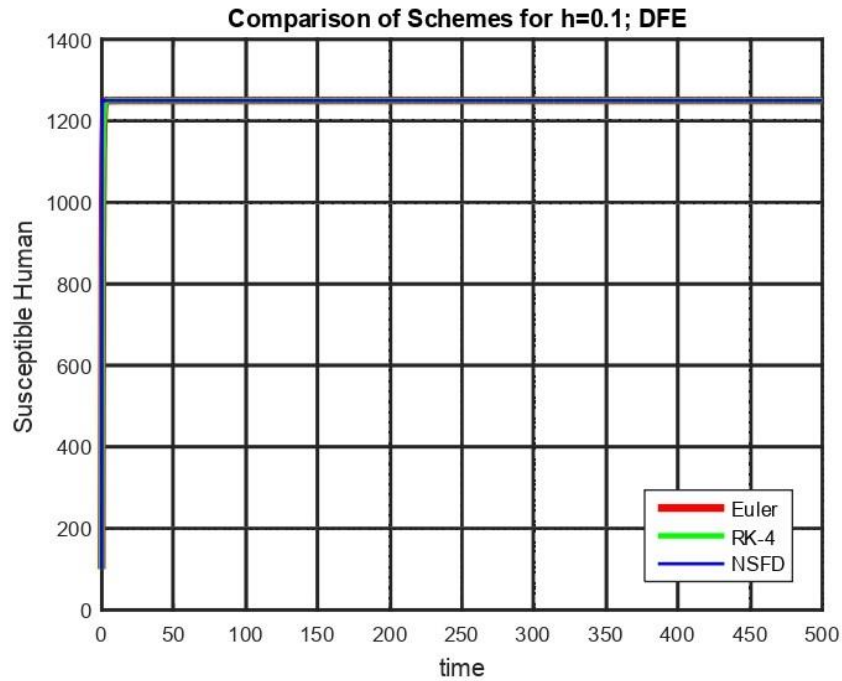
The following parameter values are used to perform numerical simulations,

*Table 1: Parameter of the Values*

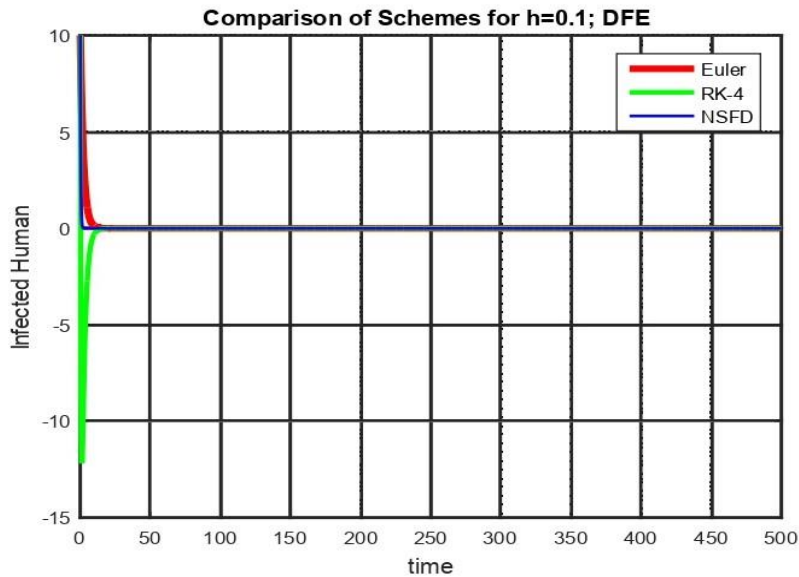
Equilibrium Points		
Parameter	DFE	EE
$\beta_s \mu_s$	5000	5000
$\mu_s$	0.2	0.2
$\mu_T$	0.2	0.2
$\mu_i$	0.5	0.5
$\mu_v$	5	5
K	0.00003	0.00003
P	100	200
$\Delta$	0.00001	0.00001
$\beta_T$	0.00003	0.00003

**Numerical Analysis of Schemes**

All numerical experiments are performed by using MAT-Lab using parameter values of table2.1.



**Fig 2: Comparison of Schemes of DFE (Susceptible Human)**



**Fig 3: Comparison of Schemes of DFE (Infected Human)**

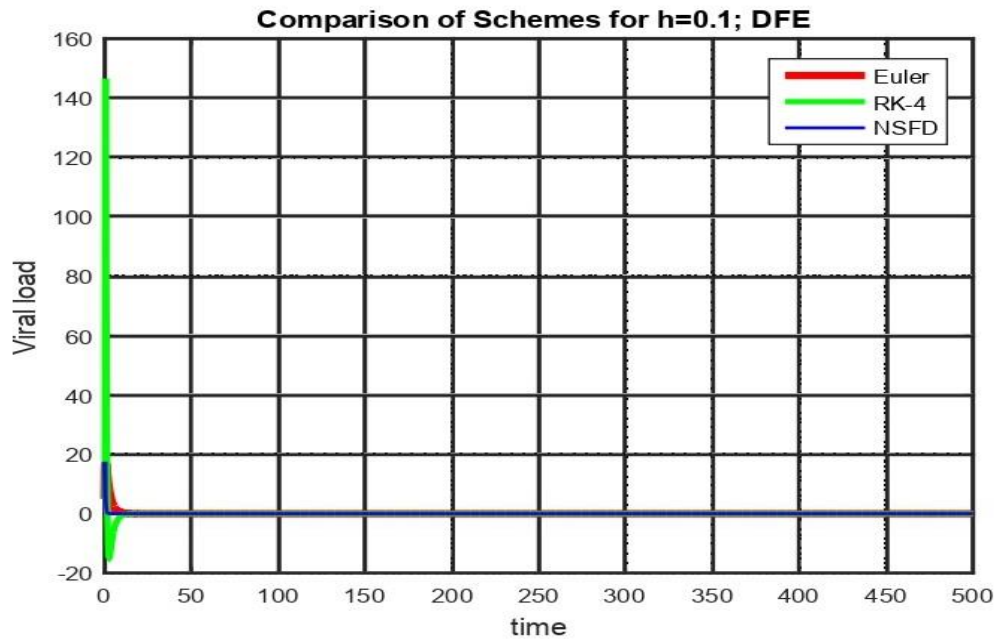


Fig 4: Comparison of Schemes of DFE (Viral Load)

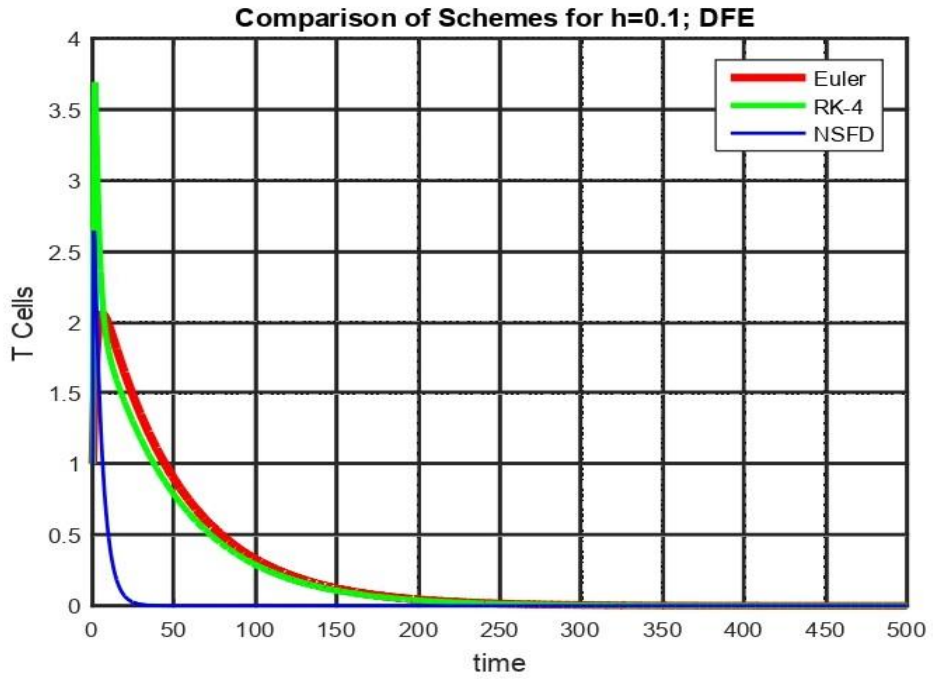


Fig 5: Comparison of Schemes of DFE (T Cells)

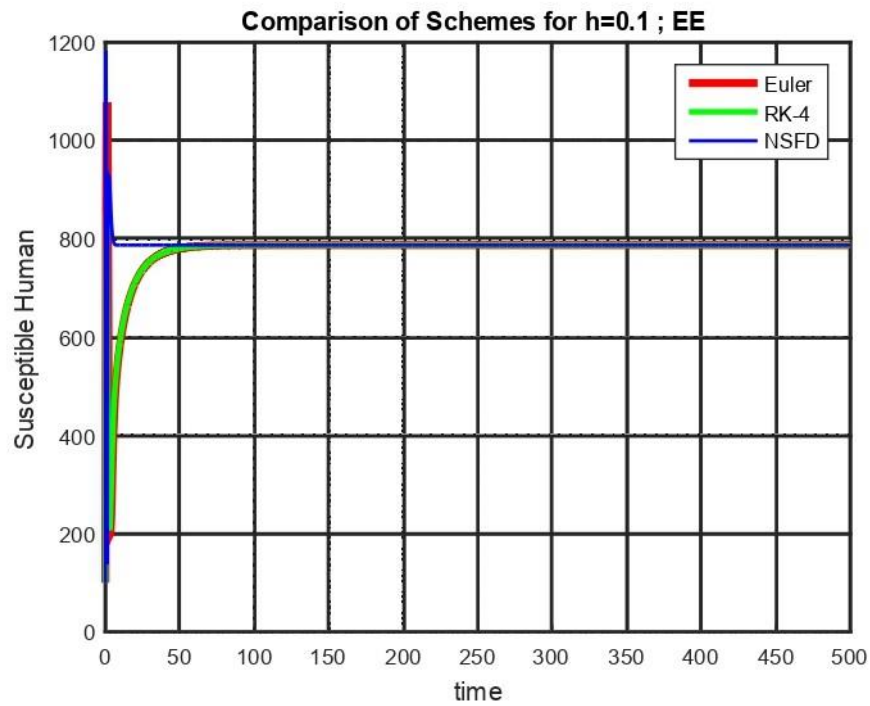


Fig 6: Comparison of Schemes of EE (Susceptible Human)

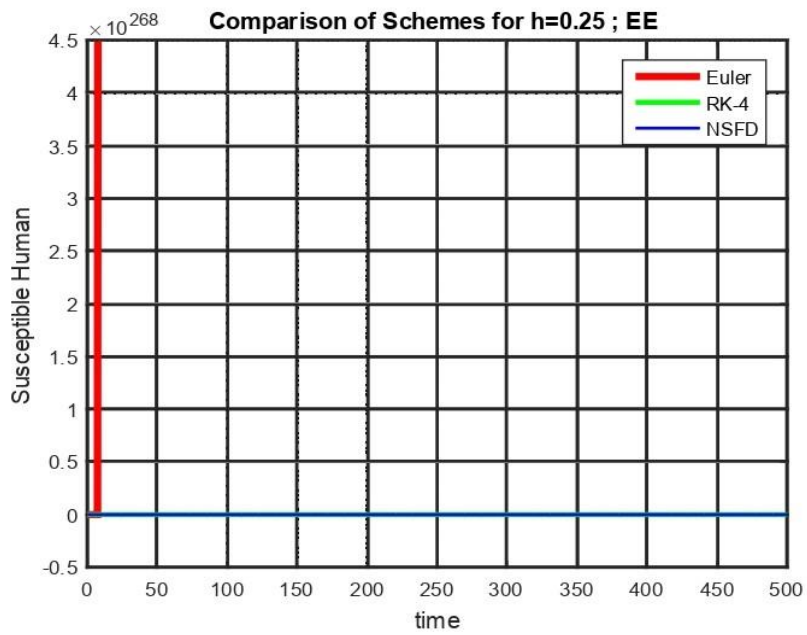


Fig 7: Comparison of Schemes of EE (Susceptible Human)

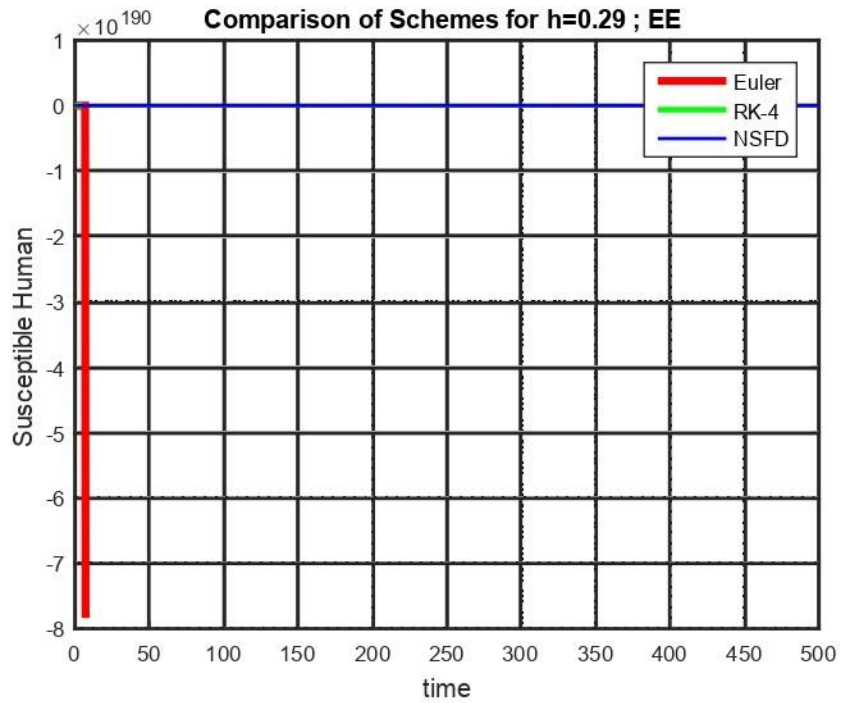


Fig 8: Comparison of Schemes of EE (SusceptibleHuman)

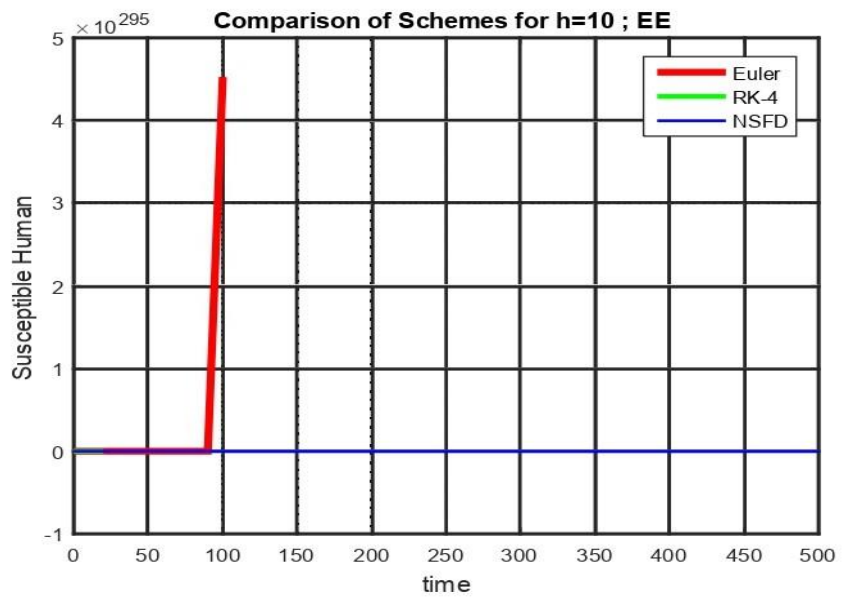


Fig 9: Comparison of Schemes of EE(Susceptible Human)



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